LEAST-SQUARES SPECTRAL ELEMENT METHODS FOR HYPERBOLIC SYSTEMS

Bart De Maerschalck, Belgium
Supervisor: Prof. H. Deconinck
Promoters: Dr. M.I. Gerritsma & Prof. P.G. Bakker (Technische Universiteit Delft)

Bona fide least squares formulations form an interesting alternative to Galerkin and Petrov-Galerkin weak formulations for the discretization of partial differential equations. The least squares methods convert well-posed partial differential equations into symmetric positive definite algebraic equations, irrespective of the type of the underlying partial differential equation. Furthermore, the least squares approach circumvents compatibility requirements in mixed/constrained formulations, which implies that no inf-sup condition between the approximating velocity space and the approximating pressure space needs to be imposed. These features allow a unified approach of a variety of flows encountered in aerospace engineering, such as compressible vs. incompressible and avoids the directional dependence in subsonic, transonic and supersonic flows.

Recently, the least squares formulation has been extended to spectral element methods and compared to the spectral Galerkin method. The convergence rate with *h*-refinement and *P*-enrichment is the same as for the Galerkin method and so is the accuracy in terms of the L2-norm and the H1-norm.

In order to achieve high order accuracy as well in space as in time, a space-time formulation has been implemented. In this formulation the temporal variable is considered as an additional spatial dimension. This means that no distinction is made between the treatment of the temporal and spatial directions and the problem is augmented with one dimension.

Spectral methods perform best when the underlying exact solution is sufficiently smooth and therefore spectral methods have mainly been used in elliptic/parabolic problems. The use of spectral methods in hyperbolic problems which allow for discontinuous solutions "traditionally has been viewed as problematic" [1], and therefore very little work has been done. In the present research the method is applied to solve linear and nonlinear advection equations with both smooth and non-smooth exact solutions. A priori error estimates and numerical results show that the scheme is stable without the necessity to add any artificial diffusion. For smooth solutions high order accuracy is demonstrated. The solution converges exponentially for *p*-type refinement, i.e. increasing the polynomial interpolation degree, while the size of the elements remains unchanged. For all other cases the convergence is algebraic (See also Figure 1).

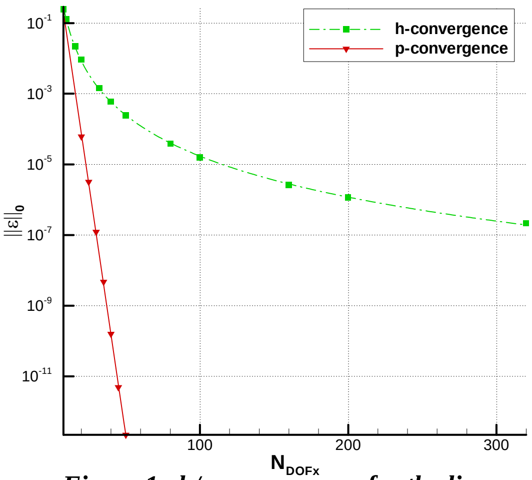

Figure 1. h/p-convergence for the linear

Recently the method has been extended to systems of non-linear hyperbolic differential equations which allow for discontinuous solution types. A conservative formulation in which the flux is added as an additional variable has been proposed. Special attention has been paid to the approximation order of the different variables of the system. So far it was globally accepted that there did not exist any *inf-sub* conditions as there are for Galerkin based methods. However, recent numerical results show that there are, specially if the underlying exact solution is non-smooth, some restrictions on how to choose the polynomial approximation order of the different variables of a hyperbolic system. Our experiences on how to choose the right finite element spaces most likely will be published in 2007.

[1] D. Gottlieb and J.S. Hesthaven: Spectral Methods for hyperbolic problems, J. Comput. Appl. Math., 128, 83-131, 2001.