DEVELOPMENT OF SOLUTION-ADAPTIVE TECHNIQUES FOR PETROV-GALERKIN METHODS IN COMPRESSIBLE FLOW

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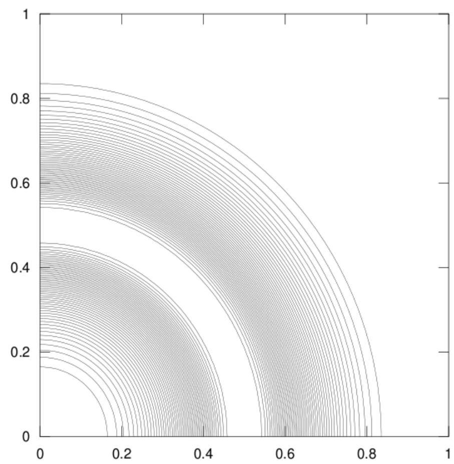
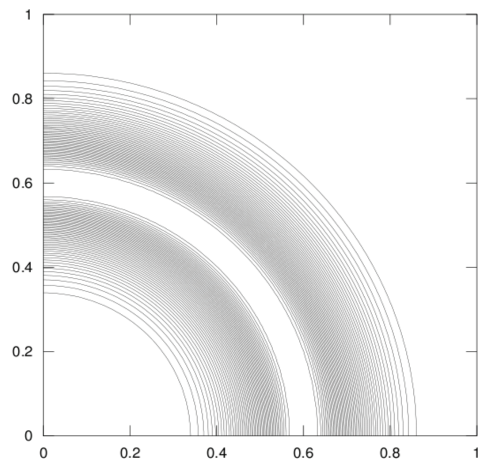
Over the last decade, much progress has been made in the area of Error Estimation. This theory provides a way to construct error indicators for CFD computations of PDE's. These can be used to drive automatic mesh adaptation algorithms. By adapting the mesh, we optimize the mesh spacings or reduce memory usage. In this field, the *a posteriori* error analysis is one of the most used procedures to compute numerical error indicators.

Becker and Rannacher worked on this issue[1,2] and they developed the so-called weighted-residual-based, or *Type I, a posteriori* error estimation. Here, the error representation formula defines the error in the target functional throught the numerical residual, weighted by the solution of an adjoint problem.
The key ingredient is right this auxiliary problem, involving the formal adjoint of the current partial differential operator. For solving the adjoint problem, added cost rises. However, it is paid back by important information which helps to identify where the real source of the error comes from. The data for the adjoint problem is a quantity of interest depending on the application. In fluid dynamics, it may be the pression at the stagnation point, the pressure-drop between inflow and outflow or the drag or lift coefficients of a body immersed into the fluid.

This procedure has been already extensively applied for some numerical methods; mostly, Discontinuous Galerkin[3] and Finite Volume[4] method. In this work, we want to apply this algorithm, for the first time, into a generic Petrov-Galerkin (PG) method, which gathers around a wide range of numerical methods: stabilised Residual Distribution (sRD), Streamline Upwind Petrov-Galerkin and bubble method, for example. A scalar hyperbolic problem is firstly used as test case and in a second moment, Euler equations will be implemented.



Primal circular problem



Adjoint circular problem

[1] R. Becker and R. Rannacher, *A feed-back approach to error control in Finite Element Methods: basis analysis and examples*, Journal Numerical Mathematics, 1996
[2] R. Becker and R. Rannacher, *An optimal control approach to a-posteriori error estimation in finite element methods*, Acta Numerica, 2001
[3] R. Hartmann, *Adaptive Finite Element methods for the compressible Euler equations*, PhD thesis in Universitat Heidelberg, 2002
[4] T. Barth, *A posteriori error estimates for higer order Godunov Finite Volume methods on unstructured meshes*, Technical Report NASA, 2002