On the dynamics of disperse sprays: high order Eulerian moment methods and dedicated realizable high order numerical methods

Adam Larat
Laboratoire EM2C, CNRS UPR 288, École Centrale Paris
Fédération de Mathématiques de l'École Centrale Paris – FR CNRS 3487
adam.larat@ecp.fr

A spray is described at a mesoscopic scale through the dynamics of a Number Density Function (NDF) f which takes its value in a large dimensional phase space $\chi = (t, \vec{\mathbf{x}}, \vec{\mathbf{v}}, S, T, \dots)$, where t and $\vec{\mathbf{x}}$ are the time and space variables, $\vec{\mathbf{v}}$ is the velocity, S and T the size and temperature of the droplets. Since χ is too large for predictive numerical simulations of the Boltzmann-like equation of the dynamics of f [1], we consider only a finite set of moments of f. To simplify the explanation, we will consider only a 7-dimensional phase space: $\chi = (t, \vec{\mathbf{x}}, \vec{\mathbf{v}}) \in \mathbb{R}^7$, and reduce the size of χ by looking at the successive moments in velocity:

$$\mathcal{M}_k = \int_{\mathbb{R}^3_{\vec{\mathbf{v}}}} f(t, \vec{\mathbf{x}}, \vec{\mathbf{v}}) \left(\otimes^k \vec{\mathbf{v}} \right) \, d\vec{\mathbf{v}}.$$

Moment of order zero is the particles density, moment of order one is the local mean momentum, moment of order two is the local mean total energy, etc. By taking successive moments of the Boltzmann-like equation cited above, we obtain a hierarchy of hyperbolic systems of conservation laws [3] which need to be closed by different physical assumption that will be discussed during the talk.

Next we will mainly focus on three main closures: the Pressure-less Gas Dynamics (PGD) which assumes the NDF is a Dirac function in the velocity direction [2] and the Gaussian and Anisotropic Gaussian (AG) models which assume the velocity repartition of the NDF is Gaussian (resp. Anisotropic Gaussian) distribution in $\vec{\mathbf{v}}$, [5, 8]. We will discuss some mathematical properties of such systems and present some features which makes their numerical approximation not really easy, especially on unstructured meshes which are mandatory for the sought complex applications. Most solutions present in fact some preferential accumulation zones with particularly high density and pressure, next to vacuum zones where ρ and p almost reach 0.

The last half of the talk will then focus on dedicated numerical method for the resolution of such conservation laws on unstructured meshes. Based on the work on Zhang&Shu [6] and Johnson&Rossmanith [9], we will present recent progresses in the preservation of convex constraints for high order numerical schemes. The preservation of such constraints is mandatory for the simulation of the moments equations since it is necessary to preserve the realizability of the set of transported moments: the fact that the considered set of moments actually corresponds to the moments

of a positive NDF f. The space of realizable moment being convex, this explains the convex constraint preservation [4, 7]. Finally numerical results will illustrate the advantage of using both higher order moments and higher order dedicated numerical schemes: it allows to capture finer physical details at reasonable computational cost.

References

[1] F.A. Williams. "Spray combustion and atomization". In: *Physics of Fluids* 1 (1958), pp. 541–545.

[2] F. Bouchut. "On zero pressure gas dynamics". In: *Advances in kinetic theory and computing*. Vol. 22. Ser. Adv. Math. Appl. Sci. World Sci. Publ., River Edge, NJ, 1994, pp. 171–190.

[3] C. D. Levermore. "Moment closure hierarchies for kinetic theories". In: *Journal of Statistical Physics* 83.5-6 (1996), pp. 1021–1065.

[4] A. Larat, M. Massot, and A. Vié. "A stable, robust and high order accurate numerical method for Eulerian simulation of spray and particle transport on unstructured meshes". In: *Annual Research Briefs 2012*. Center for Turbulence Research, NASA Ames/Stanford University, USA, 2012, pp. 205–216.

[5] F. Laurent, A. Vié, C. Chalons, R.O. Fox, and M. Massot. "A hierarchy of Eulerian models for trajectory crossing in particle-laden turbulent flows over a wide range of Stokes numbers". In: *Annual Research Briefs 2012*. Center for Turbulence Research, NASA Ames/Stanford University, USA, 2012, pp. 193–204.

[6] X. Zhang, Y. Xia, and C.-W. Shu. "Maximum-Principle-Satisfying and Positivity-Preserving High Order Discontinuous Galerkin Schemes for Conservation Laws on Triangular Meshes". In: *J. Sci. Comput.* 50.1 (Jan. 2012), pp. 29–62.

[7] M. Sabat, A. Larat, A. Vié, and M. Massot. "On the development of high order realizable schemes for the Eulerian simulation of disperse phase flows on unstructured grids: a convex-state preserving Discontinuous Galerkin method". In: *JCMF (Accepted)* (2014).

[8] A. Vié, F. Doisneau, and M. Massot. "On the Anisotropic Gaussian closure for the prediction of inertial-particle laden flows". In: *CICP* (2014). http://hal.archives-ouvertes.fr/hal-00912319.

[9] E. A. Johnson and J. A. Rossmanith. "Outflow Positivity Limiting for Hyperbolic Conservation Laws. Part I: Framework and Recipe". submitted Dec 19, 2012, `http://arxiv.org/abs/1212.4695v1`.