DEVELOPMENT OF A FINITE-ELEMENT/SPECTRAL INCOMPRESSIBLE NAVIER-STOKES SOLVER FOR APPLICATION TO LES

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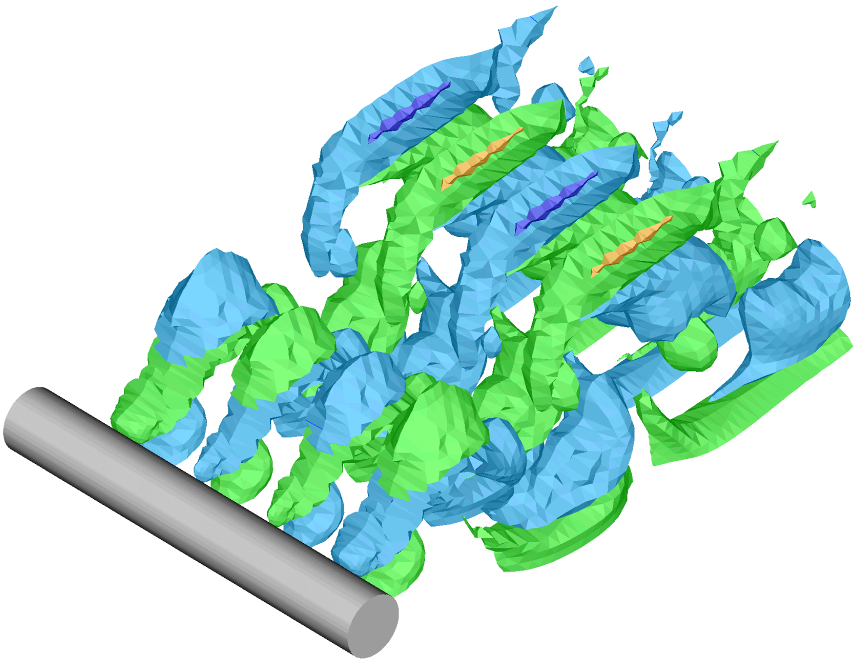
A 3D, laminar Navier-Stokes solution algorithm is developed and implemented. The algorithm, which is to be extended in the future to perform LES, assumes 2D geometry and periodic flow in the transverse direction. With these assumptions, a combined finite-element/spectral discretization is well suited, with the in-plane discretization being finite-element and the transverse discretization being spectral. This discretization technique, along with explicit treatment of the convective terms, results in a transformation of the 3D problem to a set of 2D problems that are completely decoupled within each time step. The algorithm is described in detail and applied to various test cases to show its validity.

The 2D finite-element portion of the solver uses a PSPG formulation with linear triangle elements, resulting in second-order spatial accuracy. Time accuracy is also second order: Crank-Nicolson discretization is used for the pressure and diffusion terms, while an explicit Adams-Bashforth scheme is used for the convective terms. This portion of the solver is validated by means of comparison with published benchmark values for two test cases: steady flow in a lid-driven cavity, and unsteady flow past a circular. The spatial accuracy is quantitatively validated by plotting the *x*- and *y*-velocity profiles through the center of the lid-driven cavity. The time accuracy is validated by calculating the Strouhal number of the vortex shedding from the circular cylinder.

The 3D combined finite-element/spectral algorithm solves for the Fourier coefficients of the primitive variables. The coupling between Fourier modes is contained in the convective terms only. Therefore, the explicit Adams-Bashforth treatment of these terms decouples the Fourier modes within each time step. Transformation of the primitive variables to and from Fourier space is accomplished via a FFT.

The 3D solver is operational, but not yet fully optimized and therefore limited to small test cases. It is consequently chosen, with the exception of one simple test case, to only qualitatively validate the code. The algorithm is applied to a case of pure diffusion in a unit cube for which an exact solution exists. Calculating the global error of the approximate solution verifies the second-order accuracy of the discretization. A steady Navier-Stokes approximation to a periodic jet in a channel is also calculated, and the results are found to be qualitatively correct. Finally, the solver is applied successfully to unsteady 3D flow past a circular cylinder.



Iso-surfaces of *w*-velocity for flow past a circular cylinder at Re = 200, showing breakdown of the 2D Karman vortex street.