CONSTRUCTION AND ANALYSIS OF COMPACT RESIDUAL DISCRETIZATIONS FOR CONSERVATIVE LAWS ON UNSTRUCTURED MESHES

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This thesis presents the construction, the analysis and the verification of compact residual discretizations for the solution of conservation laws on unstructured meshes. The schemes considered belong to the class of residual distribution (RD) or .fluctuation splitting (FS) schemes. The methodology presented relies on three main elements

1. Construction of compact linear first-order stable schemes for linear hyperbolic PDEs;
2. A *positivity preserving* procedure mapping stable first-order linear schemes onto nonlinear second-order schemes with non-oscillatory shock capturing capabilities;
3. A conservative formulation enabling to extend the schemes to nonlinear *CL*s.

These three design steps and the underlying theoretical tools are discussed in depth. The nonlinear *RD* schemes resulting from this construction are tested on a large set of problems involving the solution of scalar models, and systems of *CL*s. This extensive verification fills the gaps left open, where no theoretical analysis can be performed. Results are presented on the Euler equations of a perfect gas, on a two-phase flow model with highly nonlinear thermodynamics and on the shallow-water equations. On irregular grids, the schemes proposed yield quite accurate and stable solutions even on very difficult computations. These results are more accurate than the ones given by *FV* and WENO schemes. Moreover, our schemes have a compact nearest-neighbor stencil. This encourages to further develop our approach, toward the design of robust very high-order schemes for complex applications. These schemes would represent a very appealing alternative, both in terms of accuracy and efficiency, to now classical FV and ENO/WENO discretizations. A better understanding of the dissipation properties of the nonlinear discretizations proposed in the thesis might lead to further improvements in efficiency rendering the schemes very competitive also with respect to very high-order *DG* schemes.

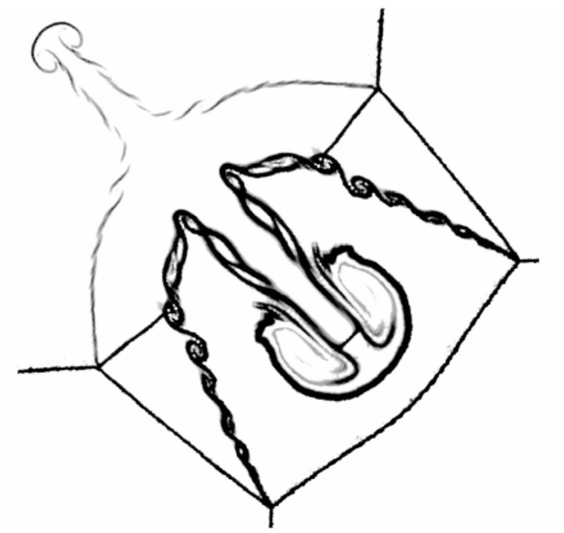

Figure 1: Instabilities in a shock/shock interaction: temperature gradient

The thesis has been completed in June 2005.