CONSERVATIVE FORMULATION OF THE MULTIDIMENSIONAL UPWIND RESIDUAL DISTRIBUTION SCHEMES FOR GENERAL NONLINEAR CONSERVATION LAWS

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In the previous years a class of multidimensional upwind fluctuation splitting or residual distribution (RD) schemes have been developed for the solution of hyperbolic systems of conservation laws on unstructured grids composed of linear finite elements. These matrix RD schemes have been used for the numerical approximation of a variety of steady flows governed by the Euler or Navier-Stokes equations.

In the framework of the RD method the existence of a robust *conservative* monotone shock capturing first order scheme is an essential building block for the development of higher order monotone nonlinear schemes. The *N* scheme is the optimal linear monotone RD scheme, which has the smallest cross-diffusion in its class. Since it operates on the quasilinear form of the governing equations, care has to be taken to keep the conservative property of this scheme at the discrete level. If the components of the flux vector can be expressed as linear or quadratic functions in terms of a properly chosen parameter vector, then exact conservation of the *N* scheme can be achieved by applying the Struijs-Roe-Deconinck (SRD) linearization. However, in several other applications such as the ideal magnetohydrodynamics (MHD) or the two-phase flow equations, the physics of the problem is described by more complex fluxes, thus precluding the simple extension of the SRD linearization.

During the last year, a new form of the system *N* scheme has been introduced to ensure discrete conservation of the fluxes for any choice of variables applied in the linearization. While the multidimensional upwinding of the convective residual is still based on the quasilinear form of the governing equations, exact conservation for flux functions of any order is incorporated via simple flux contour integrals along element boundaries, such that the underlying robustness and accuracy of the basic schemes is retained. These schemes are referred to as *Contour integration based Residual Distribution* or CRD schemes.

On (figure 1) the computation of a rotated MHD Riemann problem is shown involving two strong shocks propagating in the opposite directions. There is no SRD linearization for the MHD equations, therefore the standard RD *N* scheme is nonconservative for this case, leading to wrong shock position and wrong shock strength (left). However, the new CRD formulation is conservative and monotone, as demonstrated by the proper shocks captured without spurious oscillations (right).

Presently, work is underway to implement the new approach in a full 3D MHD solver, which was developed over the previous years.

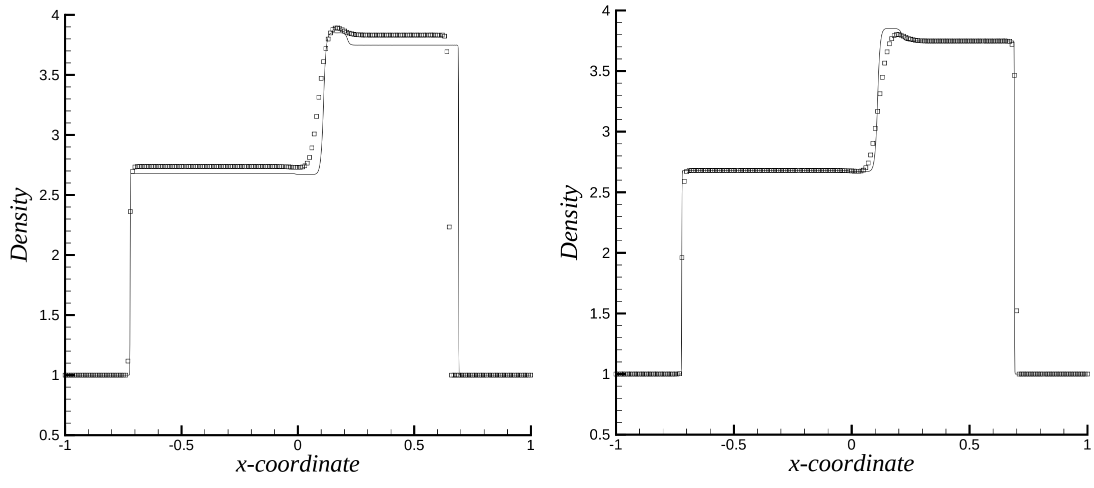

Figure 1: Solution of a rotated MHD Riemann problem. Solid line shows the exact solution, while symbols correspond to the computed values.
Left: standard RD N scheme, which is nonconservative for the MHD equations.
Right: new, conservative CRD N scheme.