AGGREGATION MULTI-GRID FOR RESIDUAL DISTRIBUTION SCHEMES

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In this report, two techniques in the field of Computational Fluid Dynamics were combined: the Residual Distribution Method (RDM) and Aggregation Multi-Grid (AggMGM). RDM is a discretization technique encompassing different Residual Distribution Schemes (RDS) on par with FEM and FVM, while AggMGM is an algebraic acceleration technique for the iterative solution of implicit systems.

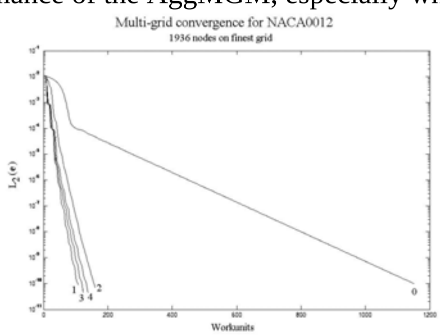
The Residual Distribution Method (RDM) has been developed as an alternative to the Finite Volume Method (FVM) and the Finite Element Methods (FEM). Different Residual Distribution Schemes (RDS) have been developed in this framework and have proven to possess considerable merit. Their main advantages are that they work on unstructured grids, have narrow stencils and allow for truly multi dimensional upwinding.

Implicit solution methods lead to a coupled system of equations. If these are solved numerically, a frequent choice is the use of iterative solvers; like Gau¼-Seidel or Jacobi. These methods reduce the high frequency errors very fast, while the low frequency errors are reduced much more slowly, and can thus be interpreted as smoothers. Multi-grid methods have been developed to speed up the process by continuously switching between fine and coarse levels, where the low frequency errors become high frequency errors (relative to the mesh size). Multi-grid can be implemented in a geometrical or algebraic way. It is in the latter category that we find the aggregation type, which groups nodes and equations into predefined aggregates, that each serve as one degree of freedom on the coarser level.

In this report we investigated if an implicit algebraic system resulting from RDS is compatible with the aggregation multi-grid (AggMGM).

The study mainly focused on two aspects: (1) The infuence of the Peclet number on the performance of the aggregation multi-grid scheme for scalar advection-diffusion equations. (2) The performance of the aggregation multi-grid for the Cauchy-Riemann equations, modelling incompressible potential flow.

It was found that for the scalar advection-diffusion equation a higher Peclet number seriously degrades the performance of the AggMGM, especially when the number of degrees of freedom is relatively low.



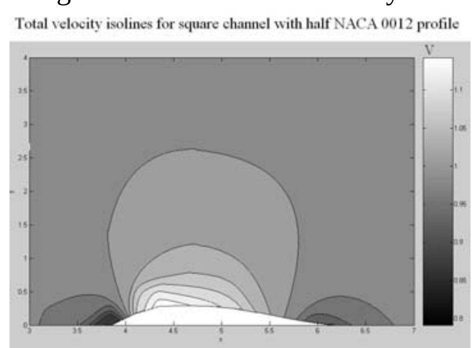


Figure 1: Tests for Cauchy-Riemann equations on NACA0012 profile at 0° AoA.
Left: Convergence of different multi-grid schemes (see text for details).
Right: Total velocity isolines around upper half of profile.

The figure above shows one of the results for the Cauchy-Riemann equations. Scheme 0 is the baseline iterative solver without multi-grid, here Symmetric Successive Under-Relaxation (SSUR). Scheme 1 is a 2-level multi-grid scheme with a direct solver on the coarsest level. Scheme 2 is similar, but with an iterative solver on the coarsest level. Schemes 3 and 4 are 3-level multi-grid schemes with a direct, resp. iterative, solver on the coarsest level. The performance is compared in number of workunits needed to obtain the required reduction of the energy norm of the error ($L_2(\mathbf{e})$), here 10^{-10}.